



Exponential Growth

The objective of this case study is to investigate how cumulative confirmed COVID-19 cases increase over time during the early outbreak phase and to quantify that relationship using nonlinear curve fitting in **Isalos Analytics Platform**. In this case study, the **Exponential growth** model is used to estimate the initial number of cases and the growth rate of the outbreak using a standard exponential growth framework.

The dataset used in this case study is derived from *the Johns Hopkins CSSE COVID-19 repository*, which contains cumulative confirmed case counts reported by date for countries and regions worldwide. For this model, the dataset was arranged so that the independent variable (X) is the number of days since the first selected observation and the dependent variable (Y) is the cumulative number of confirmed cases.

The **Exponential growth** model describes a nonlinear relationship in which the response increases proportionally to its current value over time. The equation used in this analysis is:

$$Y = Y_0 \cdot \exp(k \cdot X)$$

In this model, **Y₀** represents the **Y** value when **X** is zero, meaning the starting number of cumulative cases at the beginning of the fitted interval, and **k** represents the exponential growth rate constant expressed in reciprocal X units. If X is measured in days, then k is expressed in inverse days.

The purpose of this analysis is to determine whether the observed outbreak data during the selected early phase are consistent with an exponential growth pattern and to convert the reported cumulative case counts into meaningful quantitative outputs. The main results obtained from the fit are **Y₀** and **k**, where **Y₀** reflects the estimated starting case count and **k** reflects the outbreak growth rate. These results are useful because they allow quantitative characterization of early epidemic expansion and support interpretation of how rapidly confirmed cases increased during the selected interval.

Isalos version used: 2.0.2

Scientific Article: [https://www.thelancet.com/journals/laninf/article/PIIS1473-3099\(22\)00434-0/fulltext](https://www.thelancet.com/journals/laninf/article/PIIS1473-3099(22)00434-0/fulltext)

Step 1: Import data from file

Right-click on the input spreadsheet panel on the left and choose “**Import from File**”. Then browse to the file containing the XY dataset for this case study and load the sheet in which each row corresponds to one day value and one cumulative confirmed-case value.

	Col1	Col2	Col3	Col4	Col5	Col6	Col7	Col8
User Header	User Row ID							
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								

- Show Spreadsheet Toolbar
- Import from File
- Import from Spreadsheet
- Import from Multiple Spreadsheets
- Adjust Spreadsheet Precision
- Export Spreadsheet Data
- Clear Spreadsheet

**(Before importing into Isalos, convert the date column into a numeric time column so that the first selected date becomes Day 0, the next date becomes Day 1, and so on. This is necessary because the Exponential growth model requires a numeric X variable representing time.)*

The data will appear on the left spreadsheet.

The screenshot shows the Isalos Analytics Platform interface. At the top is a blue header with the logo and navigation menus: File, Edit, Data Transformation, Analytics, Statistics, DOE. Below the header is an orange toolbar with icons for file operations. The main workspace is divided into two panels. On the left is a spreadsheet with the following data:

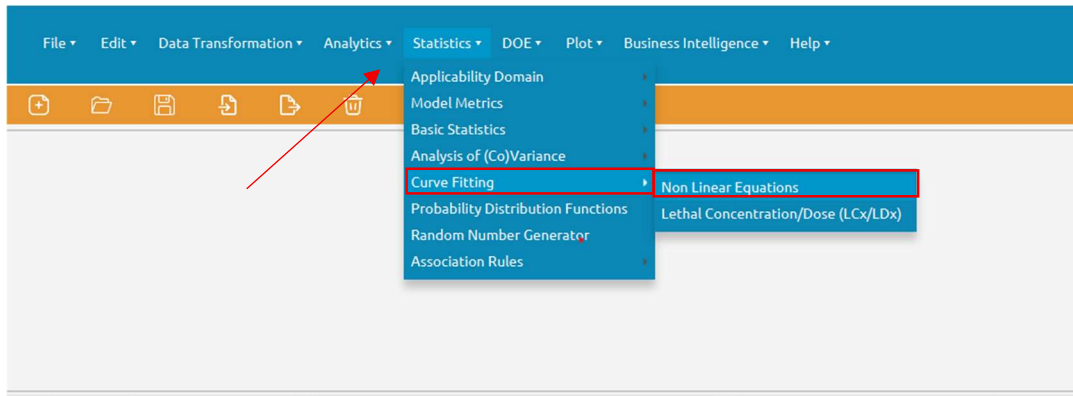
User Header	Col1	Col2 (I)	Col3 (I)	Col4	Col5	Col6
	User Row ID	Day	Cumulative_Cases			
1		0	62			
2		1	155			
3		2	229			
4		3	322			
5		4	453			
6		5	655			
7		6	888			
8		7	1128			
9		8	1694			
10		9	2036			
11		10	2502			
12		11	3089			
13		12	3858			
14		13	4636			
15		14	5883			
16		15	7375			
17		16	9172			

On the right is a configuration panel titled "Curve Fitting - Non Linear Eq". It includes the following settings:

- Category: Growth, Exponential
- Model: Exponential growth
- Independent Variable: Col2 - day
- Dependent Variable: Col3 - Cumulative
- Confidence Level (%): 95
- Confidence Interval Type: Symmetrical, Approximate
- Logarithmize Independent Variable Data
- Buttons: Show Chart, Execute, Reconfigure

Step 2: Select the desired statistical analysis model

From the toolbar, open the **Statistics** drop-down list and navigate through: **Statistics > Curve Fitting > Non Linear Equations**.



In the category list, select the appropriate category containing growth equations and then choose the **Exponential growth** model from the model menu.

Curve Fitting - Non Linear Equations

Category: Growth Equations

Model: Exponential growth

Independent Variable: []

Dependent Variable: []

Confidence Level (%): Double (0,100), Default: 95

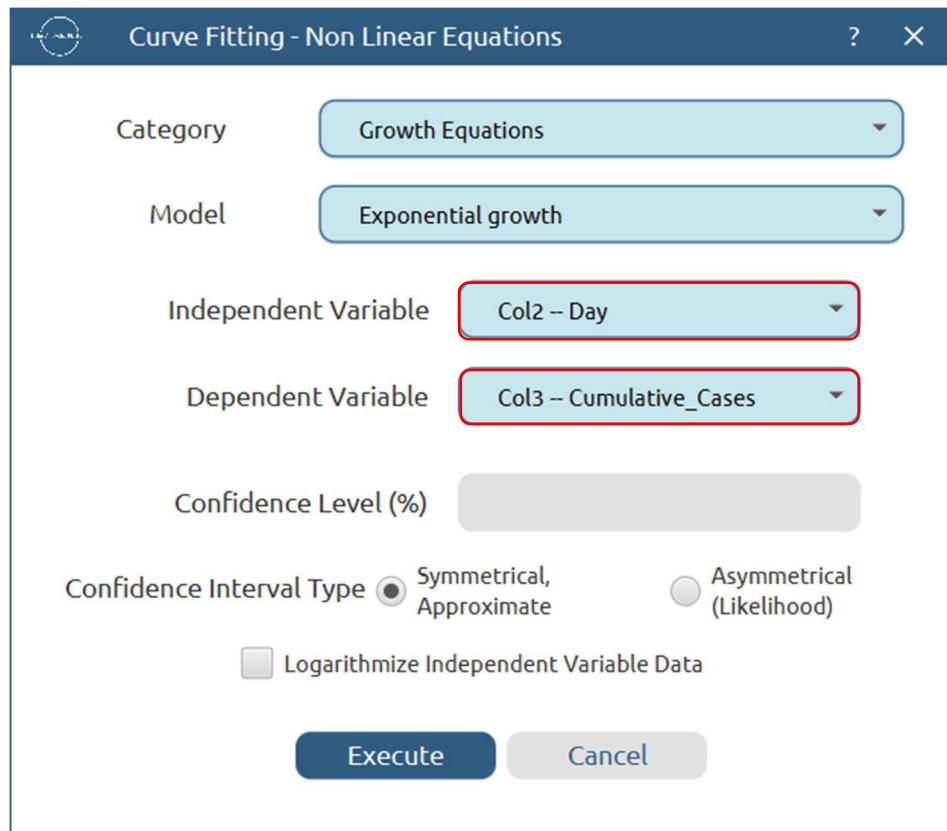
Confidence Interval Type: Symmetrical, Approximate Asymmetrical (Likelihood)

Logarithmize Independent Variable Data

Execute Cancel

Step 3: Configure variables and confidence intervals

Set the column containing the **day values** as the **independent variable (X)**, and set the column containing **the cumulative confirmed-case** values as the **dependent variable (Y)**. Set the confidence level to 95% and choose Symmetrical Approximate as the confidence interval type.



The screenshot shows a dialog box titled "Curve Fitting - Non Linear Equations". It contains the following settings:

- Category: Growth Equations
- Model: Exponential growth
- Independent Variable: Col2 -- Day
- Dependent Variable: Col3 -- Cumulative_Cases
- Confidence Level (%): 95
- Confidence Interval Type: Symmetrical, Approximate; Asymmetrical (Likelihood)
- Logarithmize Independent Variable Data
- Buttons: Execute, Cancel

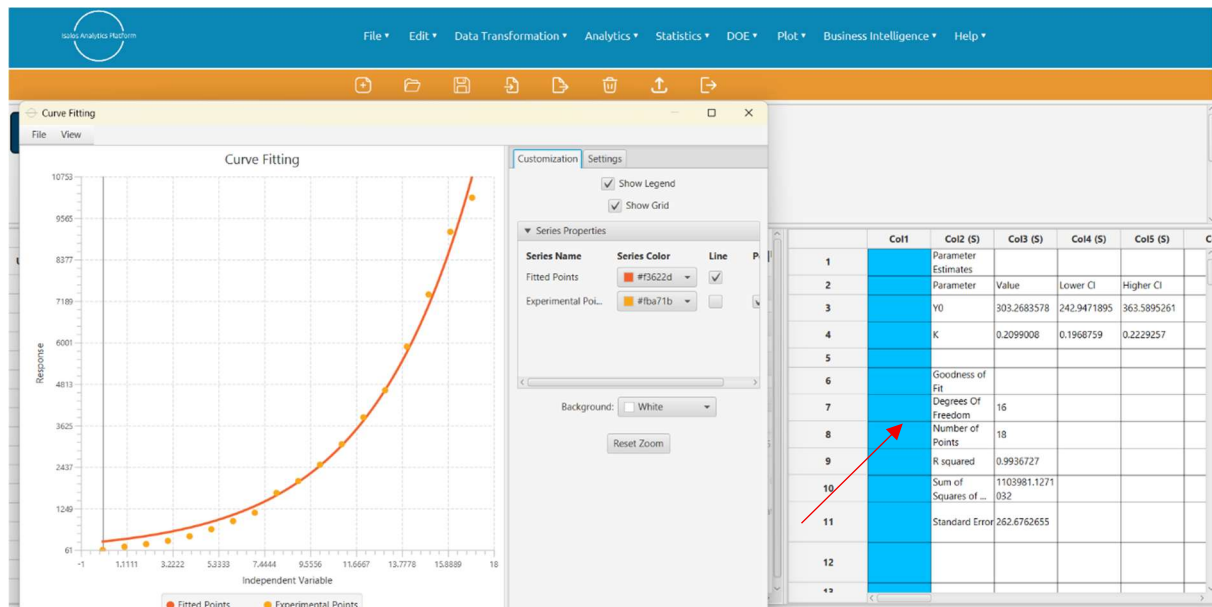
Step 4: Analyze the output and fitted curve

Once the analysis is completed, Isalos presents the fitted exponential curve together with the experimental data points, allowing direct visual assessment of how well the model captures the rise in cumulative confirmed cases over time. In this study, the graph shows an upward-curving pattern characteristic of **exponential increase** during the selected early outbreak interval.

The results page presents the estimated model parameters with their confidence limits, together with goodness-of-fit statistics and the corresponding fitted plot. In the **Exponential growth model**, the main fitted parameters are **Y0** and **k**, and these values collectively determine the starting level and the steepness of the fitted growth curve.

The **Goodness of Fit** section of the table summarizes key fitting statistics such as **the number of data points** used, **degrees of freedom**, **residual sum of squares**, and the **standard error of**

the regression. These outputs should be interpreted together with the fitted plot in order to evaluate how well the model describes the data and how reliable the estimated parameters are.



The fitted results indicate that the cumulative COVID-19 case data are well described by an **Exponential growth model**, showing that the increase in confirmed cases over the selected early outbreak interval follows a continuously accelerating nonlinear pattern rather than a linear trend. The estimated Y_0 of **303.268** represents the model-estimated number of cumulative confirmed cases at the starting point of the fitted interval, while the estimated K of **0.2099** per day represents the exponential growth rate constant that governs how rapidly the cases increase over time. Based on this growth rate, the implied doubling time is approximately **3.30 days**, indicating a rapid expansion of cases during the analyzed period.

The confidence intervals for both fitted parameters are relatively narrow and do not indicate excessive uncertainty, which supports the stability and reliability of the fitted model. The confidence interval for Y_0 ranges from **242.947 to 363.590**, showing that the estimated starting level is reasonably well constrained, while the confidence interval for K ranges from **0.19688 to 0.22293** per day, indicating a precise estimate of the outbreak growth rate over the fitted interval.

Together with the high R^2 value of **0.99367** and the fitting statistics obtained from the data points, the results indicate that the **Exponential growth model** provides a good description of the selected early-phase outbreak data. The residual error values, show that some deviation from the fitted curve remains, but overall the agreement between the observed data points and the fitted exponential trend is strong, supporting the use of this model for characterizing the early epidemic growth pattern in this study.

References:

- (1) Dong, E., Ratcliff, J., Goyea, T.D., Katz, A., Lau, R., Ng, T.K., Garcia, B., Bolt, E., Prata, S., Zhang, D. and Murray, R.C., 2022. The Johns Hopkins University Center for Systems Science and Engineering COVID-19 Dashboard: data collection process, challenges faced, and lessons learned. *The lancet infectious diseases*, 22(12), pp.e370-e376.